Mathematical Modeling in Object Localization Based on Signal Strength

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Abstract

Object localization is used on a daily basis in the form of GPS use. Examples of indoor applications of positioning systems can be found in the care of the elderly, indoor navigation apps for museum visitors… When modeling the distance between sender and receiver in localization problems in wireless communication, the dependency of the received signal strength can be expressed mathematically by an exponential function. Therefore the distance is commonly expressed in a lin-log scale as a function of the received signal strength. For engineering students it is more than interesting to see that the discipline of mathematics is at the service of a typical engineering field as telecommunication. Moreover this STEM topic is useful to make students reflect on the error of the estimated parameters when using the technique of linear regression to model the distance based on measured data obtained from a real-life test bed. The measured quantity is the received power expressed in dBm. Localization algorithms are close enough to the real world of students and interesting enough form mathematical point of view to use it in STEM classes.

Keywords: Curve fitting, telecommunication, estimation, logarithm

1. Introduction

Students are familiar with object localization in daily life in the form of GPS (Fig. 1). The underlying technical background is interesting to understand the influencing factors. This paper describes a project adequate for high school students interested in engineering themes. Section 2 explains the essential physics of wireless communication. Section 3 describes in a concise way the mathematical tools such as linear regression and logarithmic scales, required to build out of data a theoretical mathematical model that explains the object position with the signal strength.

Fig. 1. Global Positioning System

2. Object localization by means of RSSI of antennas

A Wireless Sensor Network (WSN) is built of spatially distributed autonomous sensors or nodes to monitor the physical environment and to transfer the data to a main location. Localization with WSN [1, 5] is an important field within communication networks and has multiple consumer indoor applications (indoor guiding of persons in complex buildings, offering location-based information). The Received Signal Strength Indicator (RSSI) is expressed in dBm and is used to measure the relative quality of a received signal to a client device. The unit dBm is used to indicate that a power ratio is expressed in decibels (dB) with reference to one milliwatt (mW). Hereby the relation between the powerratio and RSSI can be expressed as

$$RSSI = 10\ log_{10} \frac{P}{1\ mW}$$

The Friis transmission equation [3] states that

$$\frac{P_r}{P_t} = G_t\ G_r\ \frac{\lambda^2}{(4\ \pi)^2\ R^2}$$

with quantities as defined in Fig. 2.

$\boldsymbol{P_r} =$	power of receiving antenna
$\boldsymbol{P_t} =$	output power of transmitting antenna
$\boldsymbol{G_r} =$	gain of receiving antenna
$\boldsymbol{G_t} =$	gain of transmitting antenna
$\boldsymbol{\lambda} =$	wavelength
$\boldsymbol{R} =$	distance between the antennas

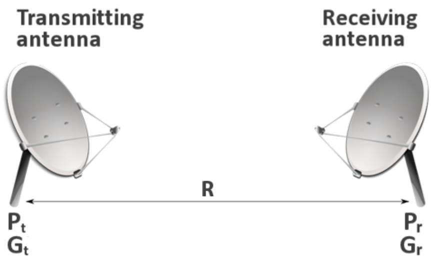

Fig. 2. Boundary conditions for the Friis transmission equation

Friis showed that in free space, RSSI degrades with the square of the distance under the assumption that the transmit antenna is omni-directional, lossless and that the receive antenna is in the far field. In practice, the actual attenuation depends on multipath propagation effects, reflections, noise, etc. In realistic models R^2 is replaced by R^n, $n=3..5$. Localization algorithms are often based on the strength of the received signal, which should be a predictor for the unknown distance.

3. Mathematical tools

As the goal is to model the dependency of the distance on the RSSI, mathematics can serve as description tool. As noise with measurements is inevitable a regression technique is of great benefit to obtain the best fitting model. Least squared differences are used to find the optimal estimates. The linear type of regression will be adequate admittedly after logarithmic transforming the data.

3.1 Lin-log scale

Inspired by the definition of RSSI, the logarithm of Friis' equation shows the linear relation between RSSI and the distance . The implies that when plotting measurement points (RSSI, distance) on a graph in a lin-log scale (Fig. 3), a linear trend will reveal.

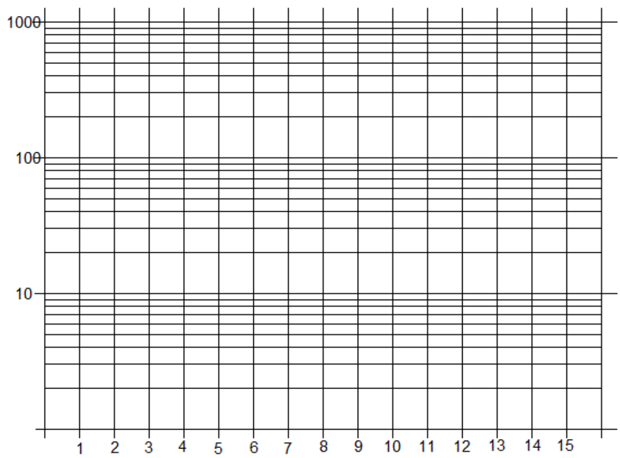

Fig. 3. Lin-log scale

3.2 Linear regression

A least-squared curve fitting method [2,4] is used to fit a model $y=f(x, \gamma_0, \gamma_1, \gamma_2,\ldots,\gamma_{k-1})$ on the basis of data points (x_i, y_i), $1 \leq i \leq n$. The parameters $\gamma_0, \gamma_1, \gamma_2,\ldots,\gamma_{k-1}$ are chosen in order to minimize the mean square error (MSE) defined by

$$\mathrm{MSE} = \frac{1}{n-k}\sum_{i=1}^{n}(y_i - \widehat{y_i})^2$$

with $\widehat{y_i} = f(x_i, \gamma_0, \gamma_1, \gamma_2, \ldots, \gamma_{k-1})$ and k is the number of estimated parameters in the model. The errors r_i in the y-direction are minimized as illustrated in Fig. 4.

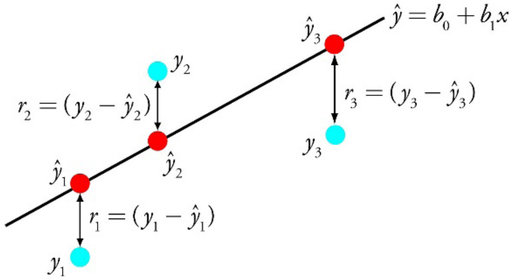

Fig. 4. Error measurement with linear regression

The regression model in the context of the antenna data uses the distance R as the outcome of RSSI as input as we want to predict the location by measuring the signal strength. Here the technique of linear regression can be applied to find estimates for the determining parameters β_0 and β_1 in the linear model

$$log_{10}R = \beta_0 + \beta_1\, RSSI.$$

RSSI	-56.9	-39.0	-54.1	-52.0	-43.0	-55.4	-55.0	-56.9	-59.6	-57.1	-61.1
R	12.1	4.1	13.0	6.0	4.8	15.4	10.2	8.7	18.8	14.6	14.3
RSSI	-57.4	-62.3	-74.7	-69.0	-58.0	-58.0	-75.0	-73.7	-79.9	-75.0	-88.0
R	19.3	19.1	23.9	26.8	24.0	28.8	31.2	29.5	35.7	34.4	40.2
RSSI	-78.7	-81.9	-91.7	-86	-89.1	-88.9	-87.9	-86.0	-84.9	-63.1	-82.0
R	44.7	43.9	49.4	48.6	67.6	58.0	54.2	53.3	52.7	22.7	39.1

Table 1. Measurement data (RSSI, distance) from a test bed

The estimation details of the parameters based on the data of Table 1, can be found in Table 2. This can be generated by statistical software such as R, SPSS, Matlab, etc. Fig. 5 and Fig. 6 show the data accompanied by the linear and non-linear regression lines respectively. Fig. 6 shows the bounds of the confidence intervals for the distance R as well.

Parameter	Estimator	95% confidence interval
β_0	−0.0256	[−0.2291, 0.1779]
β_1	−0.0202	[−0.0231, −0.01734]

Table 2. Parameter estimations for the linear regression model describing the distance as a function of the RSSI.

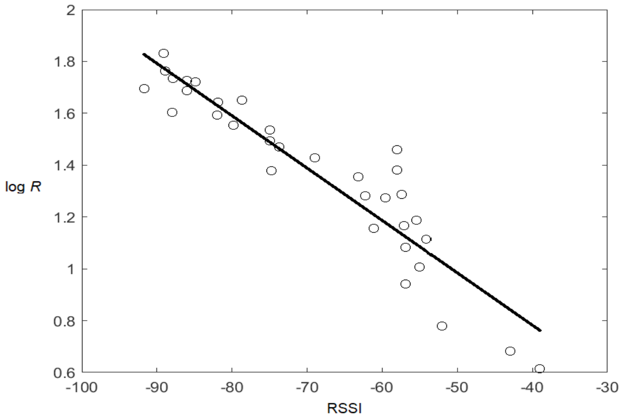

Fig. 5. Linear model for log-transformed distance versus RSSI obtained by the linear regression technique

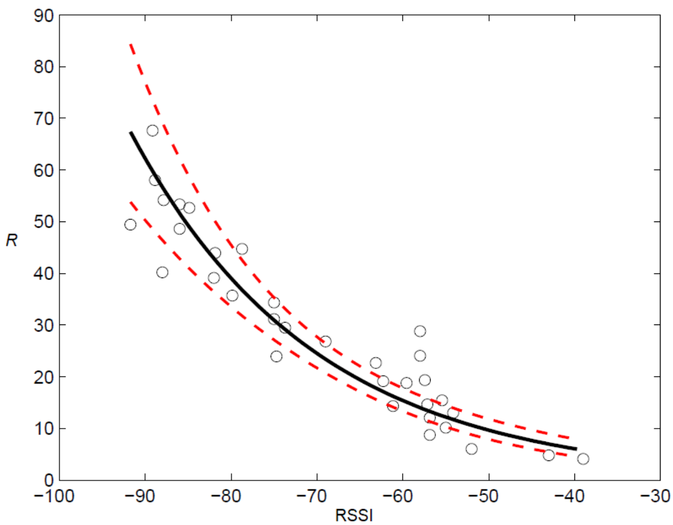

Fig. 6. Non-linear model and confidence bound for distance R versus RSSI obtained by the linear regression technique applied on $\log_{10}$ transformed distance

The coefficient of determination [4] of the regression model reaches the value of 86.86%, which guarantees its quality. The confidence bounds in Fig. 6 are wider for small values of RSSI due to the log_{10} transform. This implies loss of accuracy for small values of RSSI.

4. Didactical Advice

Before reaching a complete understanding of the connection between signal strength and object localization, teachers need to guide students through several intermediate stages. Important goals are:

- Getting students aware that $log_{10}R$ is related to RSSI in a linear way. This can clarified by taking the log_{10} of Friis' equation. Hereby the properties of logarithmic functions

$$log_{10}(ab) = log_{10}(a) + log_{10}(b)$$

$$log_{10}\left(a^b\right) = b\ \ log_{10}(a)$$

$$log_{10}\left(\frac{a}{b}\right) = log_{10}(a) - log_{10}(b)$$

can be used. Taking into account the definition of RSSI, the linear relation is visible.

- Getting students aware that a line appears when plotting an exponential function in a lin-log scale. The site https://www.desmos.com/calculator/toms4x34af is useful to let students experiment with plotting exponential functions in different scales (lin-lin, lin-log, log-log) to achieve deeper understanding of logarithmic transformations.

Critical questions can also contribute to reach the level of deeper understanding, such as:

- Why do we have negative RSSI values?
- Will the RSSI be lower or higher when the signal becomes better, i.e., less loss between sending and receiving ?
- Can you explain why there is a decreasing trend when plotting the distance between sending antenna and receiving antenna in terms of the RSSI?

5. Conclusions

Modeling the distance between sender and receiver in localization problems in wireless communication, is a challenging subject for high school students interested in mathematics and applied sciences. Teachers can use the well-known GPS as starting point for the subject of object localization. From mathematical point of view the dependency of the received signal strength, expressed by an exponential function, can bring the logarithmic function and the lin-log scale under the footlight. This topic shows engineering students that the discipline of mathematics is at the service of a typical engineering field as telecommunication. Moreover this STEM topic combines a theoretical and a practical approach as measurements can be connected to a physical model. This can be carried out by students starting from data obtained from a real-life test bed.

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